Abstract of Ph.D. thesis titled

Non-convex Optimization Methods for Enhanced Performance for Problems in Graph Signal Processing, Phase-Retrieval and Sensing

Ghania Fatima, 2020CRZ8529

The complexity of optimization problems varies based on the nature of the objective function, constraints, and structural properties such as sparsity and rank deficiency. While convex optimization offers well-established solution techniques with strong theoretical guarantees, many real-world problems are inherently non-convex solving which remains a significant challenge, making non-convex optimization an active area of research in science and engineering. This thesis aims to develop efficient algorithms for solving non-convex problems across three research areas: graph signal processing, phase-retrieval, and sensing, primarily leveraging and extending classical methods like block coordinate descent (BCD) and majorization-minimization (MM).

In the area of **graph signal processing**, this thesis first proposes an MM-based algorithm to learn a sparse weighted graph by estimating its adjacency matrix under the assumption that observed signals vary smoothly over the graph nodes. Notably, the proposed algorithm requires no hyperparameter tuning and features automatic elimination of inactive variables during iterations, enhancing computational efficiency. Next, the thesis addresses the challenge of learning graphs in the presence of outliers by maximizing the penalized log-likelihood of the uncorrupted data, selecting the penalty using the false discovery rate (FDR) principle, while simultaneously estimating the number and locations of outliers and the precision matrix under graph Laplacian constraints. The thesis also considers covariance matrix estimation under positivity and sparsity constraints. It first introduces a BCD-based maximum likelihood estimation method for estimating covariance matrices with non-negative off-diagonal elements, which has applications to portfolio selection in finance. Next, a two-stage procedure is proposed for estimating sparse covariance matrices. First, the sparsity pattern of the target covariance matrix is estimated using an FDR-based multiple-hypothesis testing method. Then, the nonzero values are estimated using either a BCD or a proximal distance approach. Lastly, the thesis presents a unified framework for estimating sparse covariance matrices by integrating sparsity pattern selection and covariance matrix estimation in a single step, eliminating the need for hyperparameter tuning. A cyclic majorization-minimization technique is introduced within this framework to solve an L0-norm penalized MLE problem.

Under **phase-retrieval**, the thesis first introduces a double-loop MM algorithm to recover the original signal from measurements that contain only the magnitude of a linear function of the unknown signal, where the measurement noise follows a Poisson distribution. This is achieved by reformulating the original minimization problem into a saddle-point problem. The thesis further extends the algorithm to handle various L1-regularized Poisson phase-retrieval problems (which exploit sparsity). Additionally, the thesis proposes an iterative algorithm for the phase-retrieval problem under the scenario when the measurements follow Poisson plus

Gaussian (PG) distribution, which is a more realistic model in applications like astronomy, microscopy, medical imaging, and remote sensing. The proposed algorithms are compared with previously proposed algorithms under different experimental settings.

In the area of **sensing**, the thesis undertakes the problems of optimal placement of sensors for improving the accuracy of various source localization techniques. First, the thesis presents an optimal sensor placement methodology, based on MM, for the hybrid localization technique by reformulating the problem into a saddle-point problem. The proposed method is designed for A-, D-, and E- optimality criteria and can work for both uncorrelated and correlated noise in the measurements. Next, the thesis proposes a numerical method for the optimal placement of the receivers in a multistatic target localization system (with a single transmitter and multiple receivers) using A- and D- optimality criteria. The proposed algorithm, based on the principle of block majorization minimization (block MM), can also handle the cases where the transmitter also acts as a receiver. Lastly, the thesis focuses on finding the optimal sensor placements for source localization without any specific assumption on the actual source position and considering just a region where the source is likely to be present, yielding a robust design. Moreover, each sensor position is constrained to lie within a pre-specified set (deployment constraint set). The proposed methodology, based on Block-MM, can also handle the case of nonuniform noise variances.

The numerical simulation analysis of all the methods proposed in the thesis over both synthetic and real-world data demonstrates their superior performance over the state-of-the-art algorithms.